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QUARK MASS CORRECTIONS TO THE Z BOSON DECAY RATES [1]

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Abstract

The results of perturbative QCD evaluation of the $\sim m_f^2/M_Z^2$ contributions to $\Gamma_{Z\to b\bar{b}}$ and $\Gamma_{Z\to hadrons}$ for the quark masses $m_f \ll M_Z$ are presented. The recent results due to the combination of renormalization group constraints and the results of several other calculations are independently confirmed by the direct computation. Some existing confusion in the literature is clarified. In addition, the calculated $O(\alpha_s^2)$ correction to the correlation function in the axial channel is a necessary ingredient for the yet uncalculated axial part of the $O(\alpha_s^3)$ mass correction to the Z decay rates. The results can be applied to the τ hadronic width.

Recent analyses show that the LEP result for the branching fraction $\Gamma_{Z\to b\bar{b}}/\Gamma_{Z\to hadrons} = 0.2208 \pm 0.0024$ [1] differs (is larger than) the Standard Model prediction [2] by 2-2.5 σ with the top mass at around 174 GeV [3]. Although the search for implications of this fact beyond the Standard Model is already began (see, e.g., [4]), a further analysis within the Standard Model is still an important issue.

Briefly, a current state of the perturbative QCD evaluation of the $\Gamma_{Z\to hadrons}$ and related quantities is as follows. The QCD contributions are evaluated to $O(\alpha_s^3)$ in the limits $m_f = 0$ ($f = u, d, s, c, b$) and $m_t \to \infty$ [5]. The leading correction to the above results due to the $O(\alpha_s^3)$ triangle anomaly type diagrams with the virtual top quark has also been calculated in the limit $m_t \to \infty$ [6]. The calculations to $O(\alpha_s^2)$ are more complete. Indeed, the electron positron annihilation R-ratio was calculated long ago [7] in the limits $m_f = 0$ ($f = u, d, s, c, b$) and $m_t \to \infty$. The corrections due to the large top-bottom mass splitting have been evaluated

[1] The part of this work has been done in collaboration with D. E. Soper.

in [8, 9] and thus the axial channel, applicable to $\Gamma_{Z\to hadrons}$, has also been validated. The $O(\alpha_s^2)$ effects of the virtual heavy quark in the decays of the Z-boson have been evaluated in [10, 11]. The same effect has been studied previously in [12] in the limit $m_t \to \infty$. The present knowledge of the high order QCD and electroweak corrections to $\Gamma_{Z\to hadrons}$ are summarized in the review article [2], providing all essential details.

The subject of the present work is the correction due to the nonvanishing "light" quark masses. The following discussion will be for the quark of flavor f ($f \neq t$) in general. However, in fact, only $f = b$ is a relevant case and the masses of u, d, s and c quarks can safely be ignored at the Z mass scale. Note that the corrections $\sim \alpha_s^2 m_b^2/M_Z^2$ for the vector and axial parts of the $Z \to b\bar{b}$ decay rate were obtained in [13] and [14] correspondingly. Those evaluations are based on an indirect approach, using the renormalization group constraints and the results of earlier calculations of the correlation functions in the vector [15, 16] and scalar channels [17]. One of the aims of the present work is to obtain the quark mass corrections to $\Gamma_{Z\to q_f\overline{q}_f}$ in both channels by a direct calculation and to check the method and the results used in [13, 14] at once. Moreover, it should be stressed that there is a disagreement between the results of [15] and [16]. [2]

The quantity R_Z is defined as the ratio of the hadronic and electronic Z widths

$$R_Z = \frac{\sum_{f=u,d,s,c,b} \Gamma_{Z\to q_f\overline{q}_f}}{\Gamma_{Z\to e^+e^-}}. \tag{1}$$

The partial decay width can be evaluated as the imaginary part

$$\Gamma_{Z\to q_f\overline{q}_f} = -\frac{1}{M_Z}\mathrm{Im}\Pi(m_f, s+i0)\Big|_{s=M_Z^2}, \tag{2}$$

where the function Π is defined through a correlation function of two flavor diagonal quark currents

$$i\int d^4x e^{iqx} < T j_\mu^f(x) j_\nu^f(0) >_0 = g_{\mu\nu}\Pi(m_f, Q^2) - Q_\mu Q_\nu \Pi'(m_f, Q^2). \tag{3}$$

Here, Q^2 is a large ($\sim -M_Z^2$) Euclidean momentum. According to the Standard Model, the neutral weak current of quark coupled to Z boson is

$$j_\mu^f = \left(\frac{G_F M_Z^2}{2\sqrt{2}}\right)^{1/2} (g_f^V \overline{q}_f \gamma_\mu q_f + g_f^A \overline{q}_f \gamma_\mu \gamma_5 q_f), \tag{4}$$

where the electroweak vector and axial couplings are defined in a standard way:

$$g_f^V = 2I_f^{(3)} - 4e_f \sin^2\Theta_W, \quad g_f^A = 2I_f^{(3)}.$$

[2]Numerically the disagreement is not large. In [18] the result of [16] was confirmed. However, as it will be shown in the present paper, the initial result of [15] turns out to be correct. Also note that unfortunately, in [13], the result of [16] was used.

The Π-function may be decomposed into vector and axial parts

$$\Pi(m_f, Q^2) = \Pi^V(m_f, Q^2) + \Pi^A(m_f, Q^2). \tag{5}$$

The Feynman diagrams that contribute in Π to $O(\alpha_s^2)$ are shown in Fig. 1.

Fig.1 Feynman diagrams contributing in the correlation function to $O(\alpha_s^2)$. The cut diagrams contribute in $\Gamma_{Z\to q_f \overline{q}_f}$ at the same order.

The effects of the last two diagrams in Fig. 1 (so called triangle anomaly type diagrams) have been studied in [8, 9] and will not be considered here. In the first diagram, the shaded bulb includes any interactions of quarks and gluons (or ghosts) allowed in QCD and the dots cover any number of gluon propagators that gives one-, two- and three-loop topologies. The crosses denote current vertices corresponding to the vector or the axial parts of the current (4).

Because the problem scale ($\sim M_Z$) is much larger than the quark masses, the following expansion is legitimate:

$$\Pi^{V/A}(m_f, m_v, Q^2) = \Pi_1^{V/A}(Q^2) + \frac{m_f^2}{Q^2}\Pi_{m_f^2}^{V/A}(Q^2) + \sum_{v=u,d,s,c,b} \frac{m_v^2}{Q^2}\Pi_{m_v^2}^{V/A}(Q^2) + \cdots \tag{6}$$

The last term in the above expansion is due to the certain topological types of three-loop diagrams containing virtual fermionic loop. The effects of the virtual top quark in the decays of the Z-boson have been studied in [10, 11, 12] and will not be discussed here. The period in eq.(6) covers the terms $\sim m_f^4/Q^4$ and higher orders. Those terms at $-Q^2 = M_Z^2$ for the Z decay are heavily suppressed and can safely be ignored.

The expansion coefficients Π_i can be calculated in a way similar to the one used in [19] for the evaluation of the fermionic decay rates of the Higgs boson. In fact, the whole calculational procedure can be combined in one equation (in the limit $m_t \to \infty$)

$$\Gamma_{Z\to q_f\overline{q}_f} = -\sum_{i=V,A}\sum_{\substack{n,k=0,1\\ n+k\le 1}} \frac{1}{(2n)!(2k)!}$$

$$\frac{1}{M_Z}\mathrm{Im}\Big\{ Z_m^{2(n+k)} m_f^{2n} m_v^{2k} \Big[\Big(\frac{d}{dm_f^B}\Big)^{2n}\Big(\frac{d}{dm_v^B}\Big)^{2k}\Pi^i(\alpha_s^B, m_f^B, m_v^B, s+i0)\Big]_{\substack{m_f^B=m_v^B=0\\ \alpha_s^B\to Z_\alpha\alpha_s}}\Big\}_{s=M_Z^2} \quad (7)$$

where B labels the unrenormalized quantities. Z_m and Z_α are the $\overline{MS}$ renormalization constants of the quark mass and the strong coupling correspondingly and can be found, for instance, in [19]. The summation over the virtual quark flavors $v = u, d, s, c, b$ is assumed in Π^i. Note that the introduction of the so called D-function (see, e.g., [13, 14]), which is the Π-function differentiated with respect of Q^2, is not necessary in this calculation. Note also that eq.(7) does not include important effects from the last two diagrams in Fig. 1 (evaluated in [8, 9]) and the virtual top quark effects (evaluated in [10, 11, 12]).

In the $\overline{MS}$ [20] analytical calculations of the one-, two-, and three-loop dimensionally regularized [21] Feynman diagrams, the FORM [22] program HEPLoops [23] is used.

For the massless limit coefficients $\Pi_1^{V/A}$ in the expansion (6), the known results are obtained. The perturbative expansion of the $\sim m_f^2$ part in the r.h.s. of eq.(6) has the form

$$\frac{m_f^2(\mu)}{Q^2}\Pi^{V/A}_{m_f^2}(\alpha_s(\mu), Q^2) + \sum_{v=u,d,s,c,b}\frac{m_v^2(\mu)}{Q^2}\Pi^{V/A}_{m_v^2}(\alpha_s(\mu), Q^2)$$
$$= \frac{G_F M_Z^2}{8\sqrt{2}\pi^2} g_f^{V/A}\sum_{i=0}^{3}\sum_{j=0}^{i+1}\Big(\frac{\alpha_s(\mu)}{\pi}\Big)^i \log^j\frac{\mu^2}{Q^2}\Big(m_f^2(\mu) d_{ij}^{V/A} + e_{ij}\sum_{v=u,d,s,c,b} m_v^2(\mu)\Big). \quad (8)$$

The coefficients e_{ij} are the same in both channels for obvious reasons. Moreover, they get nonzero values starting at the three-loop level ($i \geq 2$). The summation index j runs from zero to $i+1$ since, in general, the maximum power of the pole that can be produced by a multiloop Feynman diagram is equal to the number of loops.

The direct computation of all relevant one-, two- and three-loop Feynman diagrams for the standard QCD with the $\mathrm{SU}_c(3)$ gauge group gives in the vector channel:

$$d_{00}^V = -6;$$
$$d_{10}^V = -16, \quad d_{11}^V = -12; \quad (9)$$
$$d_{20}^V = -\frac{19691}{72} - \frac{124}{9}\zeta(3) + \frac{1045}{9}\zeta(5) + N_f\frac{95}{12}, \quad d_{21}^V = -\frac{253}{2} + N_f\frac{13}{3}, \quad d_{22}^V = -\frac{57}{2} + N_f;$$
$$e_{00} = e_{1j} = 0, \quad e_{20} = \frac{32}{3} - 8\zeta(3), \quad e_{21} = e_{22} = 0.$$

Note that $d^V_{i,i+1} = e_{i,i+1} = 0$, because the highest poles cancel at each order after the summation of Feynman graphs within each gauge invariant set. This is the consequence of the conservation of current. The above results fully confirm the findings of [15] (see also [24]). On the other hand, the $\zeta(3)$ coefficient in d_{20} disagrees with the incorrect one presented in [16]. [3] Fortunately, the numerical difference is small. The error in [16] is due to the misprint in the $\zeta(3)$ term in the result for the three-loop nonplanar type diagram.

It can be shown that the vector part of the l.h.s. of eq.(8) is invariant under the renormalization group transformations and obeys the renormalization group equation

$$\left(\mu^2\frac{\partial}{\partial\mu^2} + \beta(\alpha_s)\alpha_s\frac{\partial}{\partial\alpha_s} - \gamma_m(\alpha_s)\sum_{l=f,v} m_l\frac{\partial}{\partial m_l}\right)[\text{vector part of the l.h.s of eq.(8)}] = 0, \qquad (10)$$

where $\beta(\alpha_s)$ and $\gamma_m(\alpha_s)$ are the QCD β-function and the quark mass anomalous dimension correspondingly. From eqs.(8,10), it is straightforward (similar to [19]) to get the $O(\alpha_s^3)$ logarithmic coefficients in the vector channel

$$\begin{aligned}
d^V_{31} &= 2(\beta_0+\gamma_0)d^V_{20} + (\beta_1+2\gamma_1)d^V_{10} + 2\gamma_2 d^V_{00},\\
d^V_{32} &= (\beta_0+\gamma_0)[2\gamma_1 d^V_{00} + (\beta_0+2\gamma_0)d^V_{10}] + (\beta_1+2\gamma_1)\gamma_0 d^V_{00},\\
d^V_{33} &= \frac{2}{3}\gamma_0(\beta_0+\gamma_0)(\beta_0+2\gamma_0)d^V_{00}, \quad d_{34} = 0,\\
e_{31} &= 2(\beta_0+\gamma_0)e_{20}, \quad e_{32} = e_{33} = e_{34} = 0.
\end{aligned} \qquad (11)$$

The known perturbative coefficients β_n and γ_n of the QCD β-function and the quark mass anomalous dimension γ_m with the proper normalization factors may be found, e.g., in [19]. The only missing coefficients at the $O(\alpha_s^3)$ in eq.(8) for the vector channel are the nonlogarithimc terms d^V_{30} and e_{30}. However, these terms have a zero imaginary part and do not contribute in the decay rate to $O(\alpha_s^3)$.

In the axial channel, the direct computation of the relevant one-, two- and three-loop Feynman graphs yields:

$$d^A_{00} = \frac{6}{\varepsilon} + 6, \quad d^A_{01} = 6;$$

$$d^A_{10} = -\frac{6}{\varepsilon^2} + \frac{5}{\varepsilon} + \frac{107}{2} - 24\zeta(3), \quad d^A_{11} = 22, \quad d^A_{12} = 6;$$

$$\begin{aligned}
d^A_{20} = \frac{19}{2\varepsilon^3} - \frac{99}{4\varepsilon^2} + \left(\frac{455}{36} - \zeta(3)\right)\frac{1}{\varepsilon} + \frac{3241}{6} - 387\zeta(3) - \frac{3}{2}\zeta(4) + 165\zeta(5)\\
-N_f\left(\frac{1}{3\varepsilon^3} - \frac{5}{6\varepsilon^2} + \frac{2}{3\varepsilon} + \frac{857}{36} - \frac{32}{3}\zeta(3)\right),
\end{aligned} \qquad (12)$$

[3] in contrary to the previous belief (see, e.g., comments in [13, 18]), that the result of [15] has been corrected in [16].

$$d^A_{21} = \frac{8221}{24} - 117\zeta(3) - N_f\Big(\frac{151}{12} - 4\zeta(3)\Big), \quad d^A_{22} = \frac{155}{2} - \frac{8}{3}N_f, \quad d^A_{23} = \frac{19}{2} - \frac{1}{3}N_f,$$

where $\varepsilon = (4-D)/2$ is the deviation of the dimension of space time from its physical value 4 within the dimensional regularization [21]. Note that the nonlogarithmic terms in d^A_{i0} contain poles, which cannot be removed by the renormalization of the quark mass and the coupling and have to be subtracted independently. However, one may not worry about those poles, since the imaginary parts of nonlogarithmic terms vanish anyway and do not contribute in the decay rate.

The above mass corrections to the three-loop correlation function of the axial vector quark currents are the new results of the present paper.

One may try to use the renormalization group arguments to obtain the $O(\alpha_s^3)$ logarithmic terms similarly to the vector channel (eq.(11)). However, to do so, the knowledge of the $O(\alpha_s^3)$ anomalous dimension is necessary along the calculated d^A_{20} coefficient. In fact, as it was discovered in [14], using the axial Ward identity, this anomalous dimension can be connected to the correlation function of the quark scalar currents which, however, is also known only to $O(\alpha_s^2)$ [17, 19].

From eqs.(6)-(9), (11) and (12), one obtains for the decay rate

$$\Gamma_{Z\to q_f\bar{q}_f} = \frac{G_F M_Z^3}{8\sqrt{2}\pi}\sum_{k=V,A} g_f^k \sum_{i=0}^{3}\sum_{j=0}^{i}\Big(\frac{\alpha_s(\mu)}{\pi}\Big)^i \log^j\frac{\mu^2}{M_Z^2}\Big(a^k_{ij} + \frac{m_f^2(\mu)}{M_Z^2}b^k_{ij} + \sum_{v=u,d,s,c,b}\frac{m_v^2(\mu)}{M_Z^2}c^k_{ij}\Big) \quad (13)$$

The massless limit coefficients $a^{V/A}_{ij}$ are calculated up to $O(\alpha_s^3)$ [5]. The coefficients b_{ij} read:

$$b^V_{00} = 0, \quad b^A_{00} = -6;$$

$$b^V_{10} = 12, \quad b^V_{11} = 0, \quad b^A_{10} = -22, \quad b^A_{11} = -12;$$

$$b^V_{20} = \frac{253}{2} - \frac{13}{3}N_f, \quad b^V_{21} = 57 - 2N_f, \quad b^V_{22} = 0, \quad (14)$$

$$b^A_{20} = -\frac{8221}{24} + 57\zeta(2) + 117\zeta(3) + N_f\Big(\frac{151}{12} - 2\zeta(2) - 4\zeta(3)\Big),\ b^A_{21} = -155 + \frac{16}{3}N_f,\ b^A_{22} = -\frac{57}{2} + N_f$$

The $O(\alpha_s^3)$ coefficients for the vector part read

$$b^V_{30} = 2522 - \frac{855}{2}\zeta(2) + \frac{310}{3}\zeta(3) - \frac{5225}{6}\zeta(5) - N_f\Big(\frac{4942}{27} - 34\zeta(2) + \frac{394}{27}\zeta(3) - \frac{1045}{27}\zeta(5)\Big) + N_f^2\Big(\frac{125}{54} - \frac{2}{3}\zeta(2)\Big) \quad (15)$$

$$b_{31} = \frac{4505}{4} - \frac{175}{2}N_f + \frac{13}{9}N_f^2, \quad b_{32} = \frac{855}{4} - 17N_f + \frac{1}{3}N_f^2, \quad b_{33} = 0.$$

The coefficients c_{ij} in both channels are

$$c_{1j} = c_{2j} = 0, \quad c_{30} = -80 + 60\zeta(3) + N_f\left(\frac{32}{9} - \frac{8}{3}\zeta(3)\right), \quad c_{31} = c_{32} = c_{33} = 0. \tag{16}$$

The evaluation of $O(\alpha_s^3)$ coefficients b^A_{3j} for the axial part requires the corresponding four-loop calculations. The $\zeta(2)$ terms in the above coefficients are due to the imaginary part of the term $\sim \log^3(\mu^2/s)$, which appears in the $O(\alpha_s^3)$ coefficients of the correlation function.

Taking $\mu = M_Z$, $N_f = 5$ and recalling the known massless limit coefficients [5], one obtains numerically

$$\begin{aligned}
\Gamma_{Z\to q_f\overline{q}_f} = \frac{G_F M_Z^3}{8\sqrt{2}\pi}\Bigg\{ & \\
& (2I_f^{(3)} - 4e_f \sin^2\Theta_W)^2 \Bigg[\left(1 + \frac{2m_f^2(M_Z)}{M_Z^2}\right)\sqrt{1 - \frac{4m_f^2(M_Z)}{M_Z^2}} \\
& \quad + \frac{\alpha_s(M_Z)}{\pi}\left(1 + 12\frac{m_f^2(M_Z)}{M_Z^2}\right) \\
& \quad + \left(\frac{\alpha_s(M_Z)}{\pi}\right)^2\left(1.4092 + 104.833\frac{m_f^2(M_Z)}{M_Z^2}\right) \\
& \quad + \left(\frac{\alpha_s(M_Z)}{\pi}\right)^3\left(-12.805 + 547.879\frac{m_f^2(M_Z)}{M_Z^2} - 6.12623\sum_{v=u,d,s,c,b}\frac{m_v^2(M_Z)}{M_Z^2}\right)\Bigg] \\
& + (2I_f^{(3)})^2\Bigg[\left(1 - \frac{4m_f^2(M_Z)}{M_Z^2}\right)^{3/2} \\
& \quad + \frac{\alpha_s(M_Z)}{\pi}\left(1 - 22\frac{m_f^2(M_Z)}{M_Z^2}\right) \\
& \quad + \left(\frac{\alpha_s(M_Z)}{\pi}\right)^2\left(1.4092 - 85.7136\frac{m_f^2(M_Z)}{M_Z^2}\right) \\
& \quad + \left(\frac{\alpha_s(M_Z)}{\pi}\right)^3\left(-12.767 + (\text{unknown})\frac{m_f^2(M_Z)}{M_Z^2} - 6.12623\sum_{v=u,d,s,c,b}\frac{m_v^2(M_Z)}{M_Z^2}\right)\Bigg]\Bigg\},
\end{aligned} \tag{17}$$

where for the Born terms, their well known exact expressions [2] are used. (These terms have once again been reevaluated here.) It should be stressed that, in order to obtain a complete (up to date) Standard Model expression for the decay rate, the following known QCD contributions should also be included: (i) The $O(\alpha_s^2)$ corrections due to the large mass splitting within the t-b doublet [8, 9]; (ii) The $O(\alpha_s^2)$ effects due to the virtual heavy quark [10, 11, 12]; (iii) The $O(\alpha_s^3)$ corrections coming from the triangle anomaly type graphs in the limit $m_t \to \infty$ [6]. One also needs to include the electroweak corrections. All those corrections can be found in [2].

The calculated quark mass corrections to $O(\alpha_s^2)$ and $O(\alpha_s^3)$ gave about 10-20% corrections to the corresponding massless results and are of marginal importance for the high precision analysis at LEP. It is reasonable to expect that the missing $O(\alpha_s^3)$ correction in the axial part will be of the order similar to the corresponding vector part result. However, the calculated mass corrections are important in the low energy analysis, e.g., at PEP and PETRA (or B-factory), where the vector part of eq.(17) is relevant.

For the Z$\to b\bar{b}$ decay mode, the $O(\alpha_s^2)$ mass corrections agree to the ones obtained in [13, 14] using an indirect approach, based on the renormalization group arguments and the results of [16, 17]. However, at the $O(\alpha_s^3)$, there is a small disagreement. This, in fact, is due to the incorrect numerical coefficient for $\zeta(3)$ term in [16], which was used in [13]. [4] In the previous equations, the strong coupling $\alpha_s(M_Z)$ and the quark mass $m_f(M_Z)$ are understood as the $\overline{MS}$ quantities renormalized at the Z mass. The relation between the $\overline{MS}$ running quark mass and the pole mass is derived from the on shell results of [25] (see [19])

$$m_f^{(N)}(\mu) = m_f\Big\{1 - \frac{\alpha_s^{(N)}(\mu)}{\pi}\Big(\frac{4}{3} + \log\frac{\mu^2}{m_f^2}\Big) - \Big(\frac{\alpha_s^{(N)}(\mu)}{\pi}\Big)^2\Big[K_f - \frac{16}{9} + \sum_{m_f<m_{f'}<\mu}\delta(m_f, m_{f'}) + \Big(\frac{157}{24} - \frac{13}{36}N\Big)\log\frac{\mu^2}{m_f^2} + \Big(\frac{7}{8} - \frac{1}{12}N\Big))\log^2\frac{\mu^2}{m_f^2}\Big]\Big\}, \quad (18)$$

where m_f is the pole mass of the quark, the superscript N indicates that the corresponding quantity is evaluated for the N active flavors of quarks, μ is an arbitrary scale. (In the case of Z decay, $\mu = M_Z$ and $N = 5$.)

$$K_f = 16.00650 - 1.04137N + \frac{4}{3}\sum_{m_l\le m_f}\Delta\Big(\frac{m_l}{m_f}\Big), \quad \delta(m_f, m_{f'}) = -1.04137 + \frac{4}{3}\Delta\Big(\frac{m_{f'}}{m_f}\Big) \quad (19)$$

and the numerical values for the Δ at the relevant quark mass ratios are given in [19]. Numerically, in the case of Z$\to b\bar{b}$ decay mode, $K_b \approx 12.5$ [5] and the sum over $m_{f'}$ drops out in eq.(18).

The calculated mass corrections to the correlation functions are relevant for the hadronic decay rates of the τ lepton.

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[4] There is also a misprint in eq.(23) for the general expression for the $O(\alpha_s^3)$ term in [13]: the division factor 92 should be replaced by 96.

[5] slightly higher than the one given in [14].

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